# GRAPHICAL AND PEDAGOGICAL ASPECTS OF NUMERICAL MODELING OF JOSEPHSON JUNCTIONS USING WOLFRAM MATHEMATICA

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***Abstract.*** *This paper presents a Mathematica-based graphical environment for the numerical investigation of Josephson junctions using advanced continuation and Newton-type methods. The application is implemented using Wolfram Mathematica and provides a user interface through which the influence of various physical parameters on the model under study can be investigated. The app integrates parameter continuation and the continuous analog of Newton's method for accurate and stable numerical solutions. A visual-interactive approach provides a deeper understanding of the phenomena described by nonlinear differential equations. Possibilities for using the application in an educational environment are presented, including in courses on nanophysics, numerical methods, applied mathematics, and computer simulations. The integration of dynamic visualization in teaching helps develop intuition and analytical thinking in students, as well as stimulates interest in scientific research in the field of superconductivity.*

**Key words:** Josephson Junctions; Numerical Modeling; Visual Learning; Wolfram Mathematica; Interactive Visualization; Nanophysics Education; Nonlinear Differential Equations; Scientific Simulation.

## 1. Introduction

Unlike previous implementations that focus mainly on numerical accuracy, this work emphasizes visualization-driven understanding and pedagogical usability in Josephson junction modeling. This work highlights how advanced computational environments like Wolfram Mathematica can be effectively leveraged in educational settings to bridge theoretical concepts with practical applications, particularly in complex fields such as nanophysics [1]. The interactive capabilities of such software facilitate a more intuitive grasp of the dynamic behavior of quantum structures, which are often described by non-linear differential equations that are challenging to solve analytically. This approach not only enhances student comprehension but also develops their analytical and problem-solving skills through direct engagement with simulations that would be impossible to perform manually [2].

## 2. Mathematical Model of the Josephson Junction

Josephson junctions [4] represent quantum systems whose physical behavior can be captured through phase-based models, making them ideal objects for computational exploration in both research and education. They are key elements in new nanotechnologies, condensed matter physics, and superconductivity due to their physical properties and wide range of applications [5]. The specific model employed in this application is chosen for its relevance to common experimental setups and its ability to capture essential nonlinear behaviors. The model incorporates parameters representing the external current, junction length, external magnetic field, and other relevant physical constants, enabling a comprehensive study of the junction's response under various conditions. Figure 1 illustrates a schematic representation of the Josephson junction. The device consists of two superconducting electrodes ($A$ and $B$) separated by a thin insulating layer ($C$).

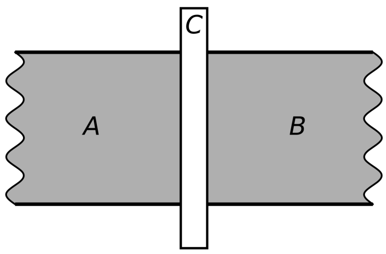


*Figure 1. Scheme of a Josephson junction [3]*

The nonlinear boundary problem that describes the model is as follows:

$$\begin{gathered} -\varphi^{''} + a_1 \sin\varphi + a_2 \sin 2\varphi - \gamma = 0, \quad x \in (-l, l), \\ \varphi^{'}(\pm l) = h_e. \end{gathered} \tag{1.1}$$

The used notations in equation (1.1) are:

1. $\varphi = \varphi(x)$ – magnetic flux,
2. $\varphi^{'} = \varphi^{'}(x)$ – internal magnetic field,
3. $l$ – half-length of the junction,
4. $h_e$ – external magnetic field,
5. $\gamma$ – external current,
6. The coefficients $a_1$ and $a_2$ reflect fabrication-dependent material properties.

## 3. Numerical Methods

The program applies two numerical methods to the mathematical model: a continuous analogue of Newton's method and an algorithm for continuation by parameters. The parameter continuation method developed by Zemlyanaya and Barashenkov [6], enables stable tracing of solution branches through bifurcation points while the continuous analog of Newton's method (Zhidkov–Puzynin approach [7]) is used to refine approximate solutions and ensure convergence even for stiff nonlinear boundary value problems. It constructs an artificial time evolution whose steady state corresponds to the desired solution, offering robust convergence for stiff boundary value problems.

## 4. Implementation in Wolfram Mathematica

The application is entirely implemented within the Wolfram Mathematica environment, using its integrated symbolic, numerical, and graphical computation capabilities. Mathematica provides a powerful platform for developing interactive interfaces and performing complex mathematical operations. The implementation involves:

- Code Structure: Organizing the numerical solvers, mathematical functions, and visualization routines into a modular and efficient structure.
- User Interface Design: Creating an intuitive graphical user interface using Mathematica's functionalities including dynamic interaction with visualized functions.
- Visualization Routines: Developing specialized functions for plotting magnetic flux, internal current and other relevant quantities.
- Data Handling: Managing the input and output data efficiently, enabling users to save results, export plots, or load predefined scenarios.

The choice of Mathematica ensures a cohesive development environment and harnesses its strengths in both computational science and visual presentation.

## 5. User interface and visualizations

The application interface is organized into two main sections: Plots and Parameters. The former contains all the plots that are visualized after solutions have been found according to the settings and parameters. These, in turn, can be set in the Parameters section.

Figure 2 shows one of the screens of the 'Plots' submenu. Visualized on it is a plot of the results for the minimal eigenvalue, which is part of the section for plots related to the varied parameter. Right next to them there is a section for the plots who are related to the x-axis. Some of them are the external current, the external magnetic field.

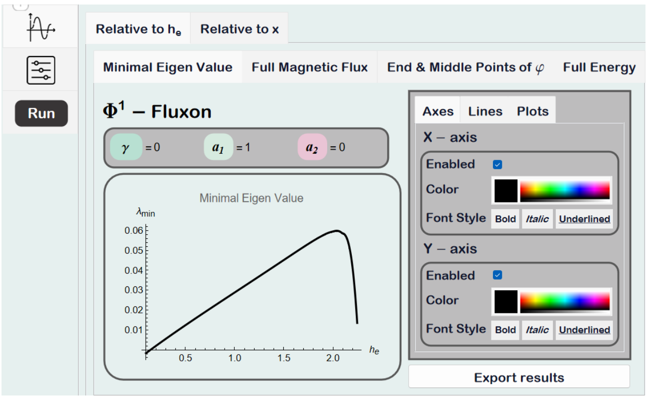


*Figure 2. 'Minimal Eigen Value' tab from the 'Plots' submenu showing the arrangement of the tabs for all of the physical characteristics. Here is shown the dependence ($\lambda_{min}$ and $h_e$) for the values $\gamma = 0$, $a_1 = 1$, $a_2 = 0$*

The Plots section is also organized using tabs. There are two sections here – one for plots against the varied parameter and one for plots against the x-axis. Each tab contains a separate plot along with other visual elements, such as a panel with graphical settings. In the Axes section, the colors of the axes can be set using a color slider.

In the Lines section, an appropriate thickness of the lines describing the displayed function can be selected using a slider. In the Plots section, the theme of the graph can be changed, as well as the background color. There are a total of five options for the theme. Below the graphical panel is the button for saving the results. When pressed, a dialog box is displayed, providing the option to save them in the form of an image, an animation, or a table file. The panel is the same in every graphics section, and the changes made are applied to all graphs.

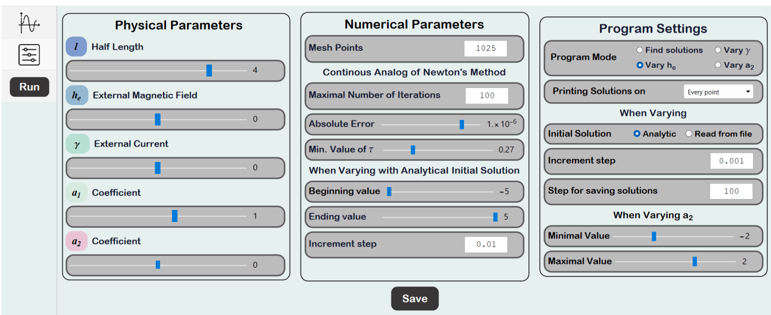


*Figure 3. A view of the 'Parameters' submenu. In this case it is chosen varying the parameter he. The other physical parameters are: $l = 4$, $h_e = 0$, $\gamma = 0$, $a_1 = 1$, $a_2 = 0$*

The 'Parameters' submenu is organized in three panels for setting the values for the physical and numerical parameters and also a panel for the settings of the program.

In the first panel, the values of the physical parameters can be set using sliders or by directly entering the values. The included parameters are the half-length of the contact, the external magnetic field, the external current, and the two coefficients designated $a_1$ and $a_2$.

The second panel contains the numerical parameters, such as the number of points on the plots, the accuracy of the final results and the minimum value of the parameter tau, which determines the iterations in the continuous analogue of Newton's method using the Ermakov-Kalitkin formula. Here, sliders are again used for some of the parameters, while input fields are used for others. Validation of the values entered by the user is implemented, and upon entering invalid values, a dialog box is displayed with a message indicating the reason for the error and the limit values of the given parameter.

The third panel contains fields for program settings. The first two fields set the program's operating mode. The next three are for the mode involving the variation of a physical parameter, and the last two are for the case of variation of the coefficient $a_2$. The selected values are saved using the "Save" button, and to start the program for finding solutions, the "Run" button is used, which is located to the left of the submenu with parameters.

The visual representation of the results and the possibility to manipulate them gives a better, easier way to learn the complex physical characteristics of the junctions.

The application allows users to dynamically manipulate key physical and numerical parameters through intuitive controls such as sliders, input fields, and checkboxes. Changes made to these parameters are reflected almost instantaneously in the graphical output, providing real-time feedback. This enables an iterative process of experimentation and observation, which is highly conducive to learning and discovery.

## 6. Pedagogical Framework and uses of the Application

The design of the application is deeply rooted in a pedagogical framework that emphasizes active learning and conceptual understanding. The primary objective of the master's thesis, from which this application originates, is to create a user-friendly, application-oriented interface for studying Josephson junctions, adaptable to other physical models with minimal variations [8]. This application aims to provide a simplified design and analysis of selected nanophysics models and is intended for both scientists and students in colleagues and universities who study Josephson junctions. The presented system has already been successfully used in the elective courses "Numerical Modeling in Nanophysics", "Visualization Methods with Wolfram Mathematica" and "Computer Simulation of Real Processes". Key elements of this framework include:

- Interactive Learning: Encouraging hands-on experimentation, where students can manipulate parameters and immediately observe the consequences on the system's behavior. This fosters an investigative approach to learning.
- Visual Representation: Prioritizing clear and informative graphical displays that translate complex mathematical solutions into intuitive visual phenomena. This helps students connect abstract equations to concrete physical manifestations.
- Conceptual Exploration: Guiding students to explore fundamental concepts such as flux quantization, phase dynamics, and nonlinear effects through guided activities or open-ended investigations.
- Problem-Solving Skills: Developing critical thinking and analytical skills by encouraging students to interpret simulated results, identify patterns, and formulate explanations.
- Connection to Theory: Explicitly linking the interactive simulations back to the theoretical models and underlying principles discussed in lectures, reinforcing the understanding of both.

This pedagogical approach is designed to transform the learning experience from passive reception of information to active engagement and discovery.

This pedagogical strategy addresses the level of difficulty of experimentally demonstrating chaotic systems, as numerical methods and visualization offer an elegant, cost-effective, and safe alternative [9]. Such interactive tools facilitate the exploration of how various physical parameters influence the system dynamics, offering insights that are often opaque through purely theoretical approaches. Specifically, modules within Wolfram Mathematica have been developed to visualize the results of such numerical studies, leveraging the system's extensive capabilities for intricate graphical representation. These modules enable the display of outcomes in various formats, tailored to the specific needs of the user, whether for detailed analysis or broad overview. This includes the dynamic plotting of solutions to demonstrate the different types of results, such as fluxon and anti-fluxon, which are some of the different types of results of Josephson junction models.

The application can be used as a pedagogical tool in various educational settings, including courses on nanophysics, applied mathematics, and computer simulations. Its visual-interactive approach directly addresses the challenges associated with teaching abstract and complex nonlinear quantum phenomena.

Through interactive exploration, students are able to develop intuition, enhance analytical thinking, visualize complex concepts and stimulate interest in scientific research.

## 7. Conclusion

This study successfully presented a Mathematica-based application specifically designed for the interactive visualization of numerical solutions in Josephson junction modeling. Through its dynamic user interface and carefully crafted pedagogical design, the tool significantly enhances the understanding of complex nonlinear quantum systems, thereby supporting both teaching and scientific research endeavors.

Future work will focus on extending the capabilities of the application. Planned enhancements include the integration of time-dependent Josephson models and comparative analysis of phase-space structures, the exploration of alternative quantum structures beyond Josephson junctions to broaden its applicability, and the development of cloud-based accessibility options. The latter would mitigate the current dependency on local Mathematica licenses and potentially improve performance for computationally intensive tasks, making the application more widely available and fostering collaborative use in diverse educational and research settings.

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