A NOVEL, MULTI-COMPONENT OBJECTIVE FUNCTION FOR MODELING REAL-WORLD VEHICLE ROUTING PROBLEM CONSTRAINTS

Nikolay Pavlov, Denis Bechiragich, Asen Rahnev

Abstract. *Standard Vehicle Routing Problem (VRP) models, often focused on minimizing total distance, fail to capture the complex, competing constraints of real-world logistics. In sectors like daily goods distribution, operational success depends on balancing route efficiency with critical business rules, such as client time-windows, variable service times, and differing client priorities.*

This work introduces a novel, multi-component objective function designed specifically to bridge this gap. Our function integrates base travel time with a flexible system of weighted penalties for service timeliness (early or late arrivals) and a scaling factor for client priority.

To demonstrate its efficacy in modeling real-world scenarios, we apply the function to guide an optimization algorithm on a test instance. We show how the resulting routes systematically balance travel time against critical business constraints in a way that simple min-distance models cannot. By adjusting the function's component weights, we demonstrate its capability to produce operationally-aware routes that verifiably prioritize key clients and respect time-windows, better reflecting the complex decision-making required in practical logistics. This research validates the function's utility as a superior model for real-world operations.

Key words: Vehicle Routing Problem, VRP, Objective Function, Real-World Constraints, Logistics Modeling, Time Windows, Client Priority, Multi-Component Model.

1. Introduction

The optimization of transportation routes is a fundamental challenge in modern logistics [4, 6]. While classical variations of the Vehicle Routing Problem (VRP) focus on minimizing fleet size or distance [5, 9], practical applications in daily goods distribution require a more comprehensive approach. Operational success depends on strict compliance with business constraints rather than simple geometric efficiency [10].

Previous studies [1, 2, 3] investigated heuristic, genetic, and tabu search approaches to routing, focusing on transportation equipment efficiency. However, a persistent challenge remains: simple cost functions often produce geo-

metrically short but operationally infeasible routes. For instance, a route might be minimal in distance but fail to respect a high-priority client's time window or a mandatory driver break. Traditional models often treat these aspects as hard constraints, introducing rigidity that fails to capture the trade-offs inherent in practical decision-making [7].

In sectors such as fast-moving consumer goods (FMCG), the problem involves heterogeneous clients and strict service intervals. The inability of standard distance-minimizing functions to natively incorporate weighted factors creates a gap between theoretical optima and operational feasibility.

This paper addresses this gap by proposing a novel multi-component objective function. A unified scalar function is formulated to transform qualitative business rules into quantitative penalties. This function integrates travel time, timeliness penalties, and priority scaling into a unified metric. To validate the semantic correctness of this function, we utilize an exact brute-force method [8] on a real-world dataset. The objective function is shown to correctly reshape the global optimum in response to constraint parameter changes, ensuring operational feasibility of the resulting routes.

2. The Multi-Component Objective Function

The primary contribution of this study is the mathematical formulation of a cost function for evaluating the transition cost between nodes. Unlike standard VRP formulations [5], our cost is a composite value derived from base travel time, timeliness penalties, and priority scaling. The cost of a single transition from node i to node j is defined as follows:

$$Cost_{hop}(i,j) = \frac{C_{travel}(i,j) + C_{timeliness}(i,j)}{1 + (P_j \cdot \beta)}$$

Where C_{travel} is the base travel cost, $C_{timeliness}$ is the penalty for time window deviations, P_j is the priority of node j, and β is a sensitivity coefficient.

2.1. Component I: Travel Time

This component serves as the foundational metric representing travel duration between nodes. It is calculated based on the travel time $t_{i,j}$ extracted from the distance matrix [11]:

$$C_{travel}(i,j) = \alpha \cdot t_{i,j}^k.$$

Where α is a scalar weight and k is a polynomial degree (usually $k = 1$).

2.2. Component II: Timeliness Penalty

Strict compliance with time windows is critical [5, 7]. To avoid complex branching logic, the penalty is formulated using a unified expression based on the maximum deviation from the specified time window $[Start_j, End_j]$:

$$C_{timeliness} = w_{early} \cdot [\max(0, Start_j - \epsilon - t_{arr})]^{k_{early}}$$
$$+ w_{late} \cdot [\max(0, t_{arr} - End_j - \epsilon)]^{k_{late}}.$$

Deviations activate either the early or the late penalty term. To enforce strict deadlines, we utilize a steep exponential penalty for lateness ($k_{late} = 5$), while early arrivals are penalized linearly ($k_{early} \approx 1.2$).

2.3. Component III: Priority Scaling

To optimize service for key clients, we introduce a priority scaling factor. In this study, we utilize a binary priority scheme where $P_j = 1$ indicates a high-priority client:

$$Factor_{priority} = 1 + (P_j \cdot \beta).$$

This mechanism introduces a prioritization effect that encourages the optimization process to schedule high-priority nodes earlier in order to reduce the accumulated route cost.

2.4. Normalization

To ensure numerical stability across different datasets, the final cost is normalized into a range [0,1]:

$$Cost_{norm} = \frac{Cost_{total} - Scale_{min}}{Scale_{max} - Scale_{min}}.$$

2.5. Parameter Selection and Sensitivity

The model parameters are selected empirically to reflect the distributor's operational policy. The travel weight α is standardized (e.g., α=1) to serve as a baseline; increasing α relative to the penalties would force the algorithm to prioritize shorter travel duration over schedule compliance. The penalty weights (w) and exponents (k) determine the strictness of constraints; k_{late} is set to a high value (k_{late}=5) to model "hard" time windows where lateness is unacceptable, whereas lowering k_{late} or w_{late} would "soften" these constraints, allowing the solver to trade off minor lateness for better geometric efficiency. Finally, the priority sensitivity coefficient β determines the impact of client hierarchy, increasing β forces high-priority clients earlier in the sequence, potentially at the cost of total travel time.

3. Validation Methodology

To verify the semantic correctness of the objective function, the stochastic nature of metaheuristics [1, 2] is eliminated by employing an exact brute-force method [8]. By calculating every possible permutation ($O(N!)$), we guarantee finding the global optimum. This ensures that changes in the optimal route are driven strictly by the objective function parameters rather than by algorithmic randomness.

Experimental Setup: The experiments utilize a real-world dataset extracted from a distribution network in Plovdiv, Bulgaria. The test instance consists of $N = 9$ nodes (1 depot, 8 clients), resulting in 40,320 permutations [8], allowing for an exhaustive search within reasonable time.

4. Computational Experiments and Results

Four experiments were conducted using operational data collected via GPS devices [11]. This telemetry provided precise coordinates and service times.

4.1. Experiment I: Baseline (No Constraint Pressure)

- Setup: Lunch window is 09:00–18:00 (no effective constraint). Priorities reflect real business data (mix of $P_j = 0$ and $P_j = 1$).

- Result: Global optimum found with Total Time: 256 min.

- Analysis: Without time pressure, the timeliness penalty was zero. The function optimized based on travel time and priority scaling, visiting the lunch node purely based on geographical efficiency.

4.2. Experiment II: Shifted Lunch Window

- Setup: Lunch window constrained to 13:15–14:15.

- Result: Global optimum shifted. Total Time: 276 min.

- Analysis: The solver moved the lunch node to the end. The function determined that a small "early arrival" penalty was mathematically cheaper than the heavy cost of waiting idly or arriving late. This demonstrates the ability to trade off travel efficiency against constraint compliance [7].

4.3. Experiment III: Early Lunch Window

- Setup: Lunch window tightened to 11:20–12:00.

- Result: Global optimum changed significantly. Total Time: 239 min.

- Analysis: The strict early window forced the lunch node to be visited much earlier to avoid severe penalties. This restructuring inadvertently

led to a more efficient overall route, confirming that the Timeliness component successfully drives global route restructuring.

4.4. Experiment IV: Priority Impact

- Setup: Time constraints from Exp. III retained. Client priorities were modified (specific clients set to $P_j = 1$).

- Result: Route structure changed. Total Time: 238 min.

- Analysis: High-priority clients were pulled to earlier positions. The Priority Scaling component successfully reduced their effective cost, creating a mathematical incentive to serve them sooner. This validates the encoding of business importance into the logic.

Table 1. Comparative summary of experimental results using brute-force algorithm

Experiment Scenario	Constraints & Setup	Total Time	Lunch Arrival Time	Key Route Characteristics
I. Baseline	No time/priority constraints. Lunch window: 09:00–18:00.	256 min	09:46 (586 min)	Pure distance minimization. Lunch node visited early based on geography.
II. Shifted Window	Lunch window constrained to 13:15–14:15.	276 min	12:16 (736 min)	Lunch node pushed to the end of the route. Algorithm chose early arrival penalty over waiting time.
III. Early Window	Lunch window tightened to 11:20–12:00.	239 min	11:13 (673 min)	Lunch node forced to middle of route. Strict constraint led to a more efficient global path.
IV. Priority Impact	Same window as III. High Priority set for Clients 4, 5, 6, 7, 8.	238 min	11:30 (690 min)	High-priority clients visited first. Lunch node arrival perfectly aligned with window (no penalty)

5. Conclusion

This paper presented a multi-component objective function for VRP that effectively models complex operational constraints [6, 10]. By integrating base travel time, polynomial timeliness penalties, and priority scaling into a unified scalar metric, the proposed approach addresses key limitations of standard distance-minimizing models [4, 5].

Using an exact brute-force validation on real GPS data [11], the results confirm that the function behaves as intended. It actively trades off travel time against penalties for lateness and rewards for serving high-priority clients [7].

The experiments demonstrated that the function correctly enforces strict time windows by restructuring the entire route sequence and successfully prioritizes key accounts. Future work will focus on integrating live traffic data and applying the proposed function to larger-scale instances using hybrid metaheuristic approaches [2, 3, 12].

Acknowledgments

References

[1] S. Monov, V. Kostadinov, N. Pavlov, Routing Transportation Equipment Using Heuristic Algorithms, *International Journal of Differential Equations and Applications*, 2022, vol. 21(1), pp. 77–84

[2] S. Monov, N. Pavlov, A. Rahnev, Improving Efficiency of Routing Transportation Equipment using Genetic Algorithms, *International Journal of Differential Equations and Applications*, 2022, vol. 21 (1), pp. 105–116

[3] S. Monov, N. Pavlov, A. Rahnev, Improving Efficiency of Routing Transportation Equipment Using Tabu Search, *International Scientific Conference IMEA*, 2022, pp. 185–194

[4] P. Toth, D. Vigo, *Vehicle Routing: Problems, Methods, and Applications, SIAM*, 2nd ed., 2014

[5] J. Cordeau, G. Laporte, M. Savelsbergh, D. Vigo, Vehicle Routing, In: Handbooks in Operations Research and Management Science, *Elsevier*, 2007, vol. 14

[6] G. Laporte, Fifty Years of Vehicle Routing, *Transportation Science*, 2009, vol. 43 (4), pp. 408–416

[7] B. Eksioglu, A. Vural, A. Reisman, Cost models for vehicle routing problems, *European Journal of Operational Research*, 2009

[8] S. Violina, Analysis of Brute Force and Branch & Bound Algorithms to solve the Traveling Salesperson Problem (TSP), *Turkish Journal of Computer and Mathematics Education*, 2021, vol. 12 (8), pp. 1226–1229

[9] E. Lawler, J. Lenstra, A. Rinnooy Kan, D. Shmoys, The Traveling Salesman Problem: A Guided Tour of Combinatorial Optimization, *Wiley*, 1985

[10] K. Tan, Y. Chew, L. Lee, The Vehicle Routing Problem: State-of-the-Art, *Applied Sciences*, 2021, vol. 11 (21), 10295

[11] OSRM Project, Open Source Routing Machine (OSRM) – High perfor-

mance routing engine, 2023, Available: `http://project-osrm.org/`

[12] Google, OR-Tools: Google Optimization Tools, 2023, Available: `https://developers.google.com/optimization`

Nikolay Pavlov[1], Denis Bechiragich[1], Asen Rahnev[1]
[1] Paisii Hilendarski University of Plovdiv,
Faculty of Mathematics and Informatics,
236 Bulgaria Blvd., 4027 Plovdiv, Bulgaria
Corresponding author: `nikolayp@uni-plovdiv.bg`